MJC PHY 05 SEM :- IV Date 17/2/25

Unit :- 02

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Linear differential equ^n :-

1. Q Solve $x \ln x \frac{dy}{dx} + y = 2 \ln x$

Solu^n :- Dividing by $\ln x$, we get

$$\frac{dy}{dx} + \frac{1}{x \ln x} y = \frac{2}{x}$$

$\therefore$ IF = integrating factor = $e^{\int \frac{dx}{x \ln x}}$

The solution therefore is

$$y \ln x = \int \frac{2}{x} \ln x \, dx + c$$

$$y \ln x = (\ln x)^2 + c$$

$$\Rightarrow y = \ln x + c (\ln x)^{-1}$$

where c is an arbitrary constant.

2. Q Solve $x \frac{dy}{dx} - y = x^2$ with $y(1) = 1$.

Solu^n :- We have

$$\frac{dy}{dx} - \frac{y}{x} = x$$

which is linear differential equ^n of first order.

$$IF = e^{\int \frac{1}{x} dx} = \frac{1}{x}$$

Hence

$\therefore$ Solution is $y \cdot \frac{1}{x} = \int x \cdot \frac{1}{x}\, dx + c$

$\Rightarrow \frac{y}{x} = x + c$

Now, $y(1) = 1$

$\Rightarrow 1 = 1 + c \Rightarrow c = 0$

Hence $y = x^2$ is the required solution, which is a parabola.

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